HJ-2387

M. A. (Mathematics) (Part - II) (External) Examination

May/June – 2018

501 - Differential Geometry & Linear Algebra

Time : 3 Hours] [Total Marks : 100

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. **Fillup strictly the details of ☛ signs on your answer book.**	Seat No. :
Name of the Examination : ☛ **M. A. (Mathematics) (Part - II) (External)**	
Name of the Subject : ☛ **501 - Differential Geometry & Linear Algebra**	
☛ Subject Code No. : 2 3 8 7 ☛ Section No. (1, 2,.....) : NIL	Student's Signature

(2) All questions are compulsory.

(3) Notations used are standard.

(4) Each question is of 20 marks.

1 (a) Let M be a subspace of R^3 described by $x_1 - x_2 - x_3 = 0$. Find projection of f=(2,3,2) on M by taking the standard inner product. **7**

(b) Given f=$(2,-2,-2,6)^T$ Find a vector S, obtained as a linear combination of vectors $v_1 = (1,1,1,1)^T$, $v_2 = (1,-1,0,0)^T$ and $v_3 = (1,1,0,-2)^T$ such that $\|f - s\|$ is minimal. **7**

(c) If M=span$\{A_1, A_2\}$ where $A_1 = (1,1,1,-1)^T$ and $A_2 = (-1,2,2,1)^T$then find basis of$M^{\perp}$. **6**

OR

1 (a) Define: Null space and Range of a linear operator. Prove that the null space and range of a linear operator are subspaces of norm space v and w respectively. **7**

(b) Let $A = [A_1, A_2, A_3, \dots, A_n]$. Let $<,>$ be the standard inner product on F^mthen thefollowing statement are equivalent **7**

I) $S = Ax$ is projection of y on M.

II) Z=x minimizes $\|y - Az\|$ where $z \in F^n$.

III) X satisfies the normal equations $A^H x = A^H y$.

(c) Define: Norm and Inner product space. Let $T: V \rightarrow W$ be a linear operator and dim(v)=n and dim(w) = m. If T is invertible then prove that m=n. 6

2 (a) Let $T(x) = x'' + 2x' + x$, Find the matrix of T with respect to basis $p = \{1, t, t^2\}$. Find N (T), R (T) and solve $T(x) = 1 + t + t^2$. 7

(b) Let $T: p_2 \rightarrow R^3$ be defined by T(f)=X$(X_0, X_1, X_2)^T$ where $x_k = \int_0^{K+1} f(t)dt$; $K=0,1,2$ then find $f(t)$. 7

(c) (a) Let A be an m x n matrix and if <, > is the standard inner product on F^m. Then prove that 6

(1) $N(A) = R(A^H)^{\perp}$

(2) $R(A) = N(A^H)^{\perp}$

(3) $r(A) = r(A^H)$

OR

2 (a) Let $T: v \rightarrow v$ be a linear operator, $\beta = \{e_k\}$ and $y = \{f_k\}$ where k=1...n be two bases for v. Let the change of basis from β to γ result in some matrix p=$[p_{ij}]$ then prove that $[T]_\gamma = p^{-1}[T]_\beta\, p$. 7

(b) If T be a matrix operator T(X) = AX with $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 3 & -1 \\ -2 & 2 & 6 \end{bmatrix}$. 7

Find a basis for $N(T)$ and its dimension. Also find $rank(T)$. Is T one to one? Onto? Justify.

(c) If $\{e_1, e_2, ..., e_n\}$ is an orthogonal set and x, y are in span $\{e_1, e_2, ..., e_n\}$ so that $x = \sum_{i=1}^{n} x_i e_i$ and $y = \sum_{i=1}^{n} y_i e_i$ for some x_i and y_i. Then prove that 6

1. The coefficients x_i is uniquely given by $x_i = \frac{\langle x, e_i \rangle}{\|e_i\|^2}$
2. $\langle x, y \rangle = \sum_{i=1}^{n} x_i \bar{y}_i \|e_i\|^2$

3 (a) Define: Torsion and Screw-Curvature. A necessary and sufficient condition that a given curve is plane curve is that $\tau = 0$ at all points. 7

(b) Derive equation to determine principal curvature at every point of the space curve. 7

(c) Derive formula for curvature and Torsion in Dot notations. 6

OR

3 (a) State and prove Frenet-Serret formula. 7

(b) The necessary and sufficient condition for a curve to be a helix is that its curvature bears a constant ratio with it torsion at any point. 7

(c) For the curve X=3t, Y=$3t^2$,Z=$2t^3$. 6
Show that $\rho = \sigma = \frac{3}{2}(1 + 2t^2)^{\frac{2}{1}}$.

4 (a) Define: Osculating circle. Find the radius and centre of osculating circle. 7

(b) Prove that envelop of family of surfaces touches every member of the family at all points of its characteristics. 7

(c) Find the envelop of the sphere 6
$(X - acos\theta)^2 + (Y - asin\theta)^2 + Z^2 = b^2$.

OR

4 (a) Define: Polar developable. Prove that edge of regression of polar developable is the centre of spherical curvature. 7

(b) Prove that XY =$(Z - t^2)^2$ is developable. But the surface xyz=a^3 is not developable. 7

(c) Define: Involutes. 6
Find the equation of curvature and torsion for involutes.

5 (a) State and prove QR factorization Theorem 7

(b) Define $T: R^3 \to R^3$,T(X)= $2(X^T e_1)e_1 - (X^T e_2)e_2 + (X^T e_3)e_3$ 7
where $e_1 = (0,1,0)^T$; $e_2 = (1,0,1)^T$; $e_3 = (1,2,1)^T$.
(i)Find matrix of T with respect to standard basis.
(ii) Find matrix of inverse of T if exists.

(c) Prove that the range and subspace of a linear operator $T: V \to W$ are the subspaces of V and W, respectivey. 6

OR

5 (a) Show that principal normal of one curve be Binormal of another curve if the relation a $(k^2 + \tau^2) = bk$ must hold where a and b are constants. 7

(b) If θ is the angle between direction curves 7

$pdu^2 + 2Qdudv + Rdv^2 = 0$ at any point (u,v) then prove that

$\tan(\theta) = \frac{2H(Q^2-PR)^{\frac{1}{2}}}{ER-2FQ+GP}$

(c) Define: Developable surface. Find the condition for the surface to be developable. 6
